Received October 1999
Revised June 2000
Accepted August 2000

Momentum and heat transfer in visco-elastic fluid flow in a porous medium over a non-isothermal stretching sheet

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Keywords *Viscoelasticity, Variable porosity, Thermoconductivity, Stress, Stretching, Boundary layers*

Abstract *Visco-elastic fluid flow and heat transfer in a porous medium over a non-isothermal stretching sheet have been investigated. The flow is influenced by linearly stretching the sheet in the presence of suction, blowing and impermeability of the wall. Thermal conductivity is considered to vary linearly with temperature. The intricate non-linear problem has been solved numerically by shooting technique with fourth order Runge-Kutta algorithm after using perturbation method. The zeroth order solutions are obtained analytically in the form of Kummer's function. An analysis has been carried out for two different cases, namely prescribed surface temperature (PST) and prescribed heat flux (PHF) to get the effect of porosity and visco-elasticity at various physical situations. The important finding is that the effect of visco-elasticity and porosity is to increase the wall temperature in case of blowing and to decrease in both the cases of suction and when the stretching sheet is impermeable.*

Introduction

The study of two-dimensional boundary layer flow over a stretching sheet has generated much interest over the years because of its numerous industrial applications such as aerodynamic extrusion of polymer sheets, continuous stretching, rolling and manufacturing plastic films and artificial fibres. Boundary layer flow occurs on moving continuous surfaces such as those of long thread between a feed roll and a wind up roll. Sakiadis (1961a; 1961b) is the first among others to discuss such flow behaviours theoretically and that is followed by the work of Crane (1970) and the experimental verification work of Tsou *et al.* (1961). There are several extensions of this problem which include consideration of more general stretching velocity (Kumaran and Ramanaiah, 1996), applications to different non-Newtonian fluids and the study of heat transfer (Dutta *et al.*, 1985; Jeng *et al.*, 1986). The situation where suction and blowing exist at the moving surface is discussed by Gupta and Gupta (1977) and Chen and Char (1988). Siddappa and Abel (1985) have presented their work

International Journal of Numerical Methods for Heat & Fluid Flow, Vol. 10 No. 8, 2000, pp. 786-801.

considering continuous surface subjected to suction. Heat transfer analysis in stagnation-point fluid flow over a flat sheet stretched with linear velocity is carried out by Chaim (1996). In this study, the thermal conductivity is assumed to vary linearly with temperature. Solution of the equation of magnetohydrodynamic visco-elastic boundary layer flow was obtained by Lawrence and Rao (1995).

The above investigations are restricted to flow behaviour and heat transfer in non-porous media. However, some metallurgical processes involve cooling of continuous strips or filaments by drawing them through a quiscent liquid. The rate of cooling can be controlled and final product of required characteristics may be achieved if strips are drawn through porous media. In view of this, the study of visco-elastic flow through porous media has gained attention in recent years by several researchers. An analysis has been carried out by Gupta and Sridhar (1985) for visco-elastic effects in non-Newtonian flow through porous medium. It has been shown that under certain circumstances the fluid undergoing an external deformation may not exhibit any shear thickening. Abel and Veena (1998) have carried out the study of visco-elastic fluid flow and heat transfer characteristics in a saturated porous medium over an impermeable stretching surface. This work takes into account the effect of frictional heating and internal heat generation/absorption in the flow.

Our present work envisages to study the visco-elastic Walters' liquid B fluid flow past a stretching sheet in a porous medium. In contrast to the work of Abel and Veena (1998) and Prasad *et al.* (2000) the variable thermal conductivity is assumed here, as this is true in some polymer solutions in the class of Walters' liquid B, and that leads to non-linearity in the boundary value problem of heat transfer. Recently, flow of non-Newtonian polymer solutions was investigated by Savvas *et al.* (1994) and it is shown that computer simulation is a powerful and accurate technique to predict flow behaviour of such solutions. Therefore, in this problem, we consider two more general cases of non-isothermal boundary conditions:

(1) surface with prescribed power law temperature distribution;

(2) surface with prescribed heat flux, varying quadratically with the distance.

In addition to this, we contemplate to study the problem having linearly stretched continuous sheet with suction/blowing. Impermeable stretching sheet, where the transverse velocity is zero at the surface, is also considered. Because of the complexity and non-linearity in the proposed problem, it has been solved numerically by shooting technique with fourth order Runge-Kutta method (Chaim, 1998) following perturbation technique. The zeroth order solutions are obtained in the form of Kummer's function. Result analysis reveals that the effect of visco-elasticity is to increase the wall temperature gradient in case of blowing and to decrease in case of suction. The porosity has also significant impact on the wall temperature gradient. Velocity profile and shear stress contour have also been drawn for various situations of porosity and visco-elasticity.

Mathematical formulation

Momentum transfer

We consider two-dimensional incompressible visco-elastic laminar flow of Walters' liquid B through a porous medium. Flow is influenced by suction/blowing due to a porous sheet as a boundary wall issuing from a thin slit at x = 0, y = 0. The sheet is then stretched in such a way that the speed at any point on the sheet becomes proportional to the distance from the origin (Figure 1). The basic boundary layer equations (Abel and Veena, 1998) for the two-dimensional steady flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y} - k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\} - \frac{v}{k'}u. \tag{2}$$

where u and v are the flow velocities in x and y-directions respectively, v the kinematic viscosity, k_0 the visco-elastic parameter and k' is the porous medium's permeability parameter.

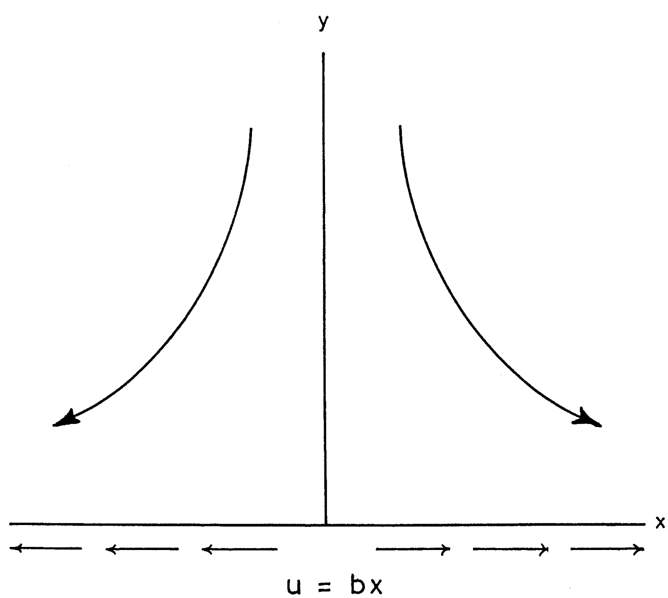

Figure 1.
A schematic diagram of the physical model

The basic boundary conditions are,

$$\begin{aligned} &u = bx \quad v = v_w \quad \text{at } y = 0, \\ &u \to 0, \quad u_y \to 0 \quad \text{as } y \to \infty \end{aligned} \tag{3}$$

We introduce the following similarity transformations

$$u = bx\, f\eta(\eta)\,, \qquad v = -\sqrt{bv}\, f(\eta) \qquad \text{and} \quad \eta = \sqrt{\frac{b}{v}}\; y \;. \tag{4}$$

With this the continuity equation (1) is identically satisfied and the momentum equation (2) reduces to the following non-linear ordinary differential equation

$$f_\eta^{\,2} - f f_{\eta\eta} = f_{\eta\eta\eta} - k_1 \left\{ 2 f_\eta f_{\eta\eta\eta} - f f_{\eta\eta\eta\eta} - f_{\eta\eta}^{\,2} \right\} - k_2 f_\eta \tag{5}$$

where $k_1 = \frac{k_o b}{v}$, the visco-elastic parameter, and $k_2 = \frac{v}{k'b}$ is the porosity parameter.

The boundary conditions take the form

$$f(0) = -\frac{v_w}{\sqrt{bv}} \qquad f_\eta(0) = 1 \quad , f_\eta(\infty) = 0 \quad \text{and} \;\; f_{\eta\eta}(\infty) = 0 \tag{6}$$

The exact solution of (5) with boundary conditions (6) is obtained as

$$f(\eta) = \frac{1 - e^{-\alpha\eta}}{\alpha} - \frac{v_w}{\sqrt{bv}}, \tag{7}$$

where α is the positive root of the cubic equation

$$\alpha^3 + \frac{1 - k_1}{v_w \frac{k_1}{\sqrt{bv}}}\alpha^2 + \frac{1}{k_1}\alpha - \frac{1 + k_2}{v_w \frac{k_1}{\sqrt{bv}}} = 0 \quad , \tag{8}$$

and it is solved by Graffe's square root method.

It should be noted that $v_w<0$ corresponds to suction, $v_w>0$ corresponds to blowing and $v_w= 0$ is the case when the stretching sheet is impermeable.

Skin friction

The shear stress at a point on the sheet is

$$\tau_0 = -\mu\left(\frac{\partial u}{\partial y}\right)_{at\; y=0}, \tag{9}$$

The non-dimensional form of shear stress is $\tau = \frac{\tau_0}{b^2 x^2 \rho}$

Heat transfer

The heat transfer in the above flow is governed by two-dimensional energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right). \tag{10}$$

where ρ is the density, c_p is the specific heat at constant pressure and k is the thermal conductivity, which is assumed to be variable here. Keeping in view that thermal conductivity k varies approximately linearly with temperature in some of the polymer solutions, we consider thermal conductivity of this problem in the form:

$$k=k_\infty(1+\varepsilon\theta) \qquad ,\theta=\frac{T-T_\infty}{T_w-T_\infty}. \tag{11}$$

where T_∞ is the constant temperature of the fluid far away from the sheet and T_w is the wall (sheet) temperature. ε is a small parameter which depends on the nature of the fluid and k_∞ is the conductivity of the fluid far away from the sheet .

The thermal boundary conditions depend on the type of heating processes through the wall surfaces under consideration. Here we consider two general cases of non-isothermal conditions:

(1) surface with prescribed power law surface temperature (PST);

(2) surface with prescribed power law heat flux (PHF).

PST case

The prescribed power law surface temperature is assumed to be a quadratic function of x and it is given by

$$T=T_w=T_\infty+A(x/l)^2, \quad \text{at } y=0 \quad \text{and } T=T_\infty \quad \text{as } y\to\infty\,; \tag{12}$$

where T_w is the variable wall temperature, A is constant and l is the characteristic length (Chen and Char, 1988). We non-dimensionalise the energy equation (10) with dimensionless temperature variables given by (11).

Considering equations (4), (7) and (11), equation (10) takes the form

$$(1+\varepsilon\,\theta(\eta))\,\theta_{\eta\eta}(\eta)\;+\;\varepsilon\,\theta_\eta^2(\eta)\;+P_r f(\eta)\,\theta_\eta(\eta)\;-2P_r f_\eta(\eta)\,\theta(\eta)\;=0. \tag{13}$$

where $P_r=\frac{\mu c_p}{k_\infty}$ denotes the Prandtl number.

The corresponding boundary conditions are

$$\theta(0)=1, \quad \theta(\infty)=0. \tag{14}$$

PHF case
The power law heat flux on the wall surface is considered to vary quadratically with distance and it is given by

$$-k\frac{\partial T}{\partial y}=D\left(\frac{x}{l}\right)^{2} \quad \text{at } y=0 \quad T=T_{\infty} \text{ as } y\rightarrow\infty \quad , \tag{15}$$

where D is another constant and l is the characteristic length.
Defining

$$g(\eta)=\frac{T-T_{\infty}}{T_{w}-T_{\infty}} \quad \text{we get} \tag{16}$$

$$T-T_{\infty}=\frac{D}{k}\left(\frac{x}{l}\right)^{2}\sqrt{\frac{v}{b}}\; g(\eta) \tag{17}$$

where $k=k_{\infty}(1+\varepsilon g)$,
Substitution of (4), (7) and (17) in the energy equation (10) leads to a non-linear ordinary differential equation of the form

$$(1+\varepsilon\, g(\eta))\, g_{\eta\eta}(\eta)+\varepsilon\, g_{\eta}{}^{2}(\eta)+P_{r}\, f(\eta)\, g_{\eta}(\eta)-2P_{r}f_{\eta}(\eta)\, g\,(\eta)=0\ , \tag{18}$$

subjected to the boundary conditions

$$g_{\eta}(0)=-1, \quad g(\infty)=0. \tag{19}$$

In the next section we solve equations (13) and (18) subjected to boundary conditions (14) and (19) respectively.

Perturbation solution

We employ perturbation technique to solve the non-linear equation (13) and (18), and so we assume

$$\theta=\theta_{0}+\varepsilon\theta_{1}+\varepsilon^{2}\theta_{2}+\ldots\ldots\ldots \tag{20}$$

$$g=g_{0}+\varepsilon g_{1}+\varepsilon^{2}g_{2}+\ldots\ldots\ldots \tag{21}$$

Using equations (20) and (21) into the equations (13), (14) and (18), (19) and equating terms with the like powers of ε, we obtain the following four boundary value problems with variable co-efficients for $\theta_0, \theta_1, g_0, g_1$ in the sequel

$$\begin{aligned}&\theta_{0\eta\eta}(\eta)+P_{r}f(\eta)\theta_{0\eta}(\eta)-2P_{r}f_{\eta}(\eta)\theta_{0}(\eta)=0\ ,\\&\theta_{0}(0)=1, \quad \theta_{0}(\eta_{\infty})=0,\end{aligned} \tag{22}$$

$$\theta_{1\eta\eta}(\eta) + \mathrm{P_r f}(\eta)\,\theta_{1\eta}(\eta) - 2\mathrm{P_r f}_\eta(\eta)\theta_1(\eta) = -\theta_{0\eta\eta}(\eta)\theta_0(\eta) - \theta^2_{0\eta}(\eta)$$
$$\theta_1(0) = 0, \quad \theta_1(\eta_\infty) = 0, \tag{23}$$

$$\mathrm{g}_{0\eta\eta}(\eta) + \mathrm{P_r f}(\eta)\mathrm{g}_{0\eta}(\eta) - 2\mathrm{P_r f}_\eta(\eta)\mathrm{g}_0\,(\eta) = 0\,,$$
$$\mathrm{g}_{0\eta}(0) = -1, \quad \mathrm{g}_{o\eta}\,(\eta_\infty) = 0\,, \tag{24}$$

$$\mathrm{g}_{1\eta\eta}(\eta) + \mathrm{P_r f}(\eta)\mathrm{g}_{1\eta}(\eta) - 2\mathrm{P_r f}_\eta(\eta)\mathrm{g}_1(\eta) = -\mathrm{g}_{0\eta\eta}(\eta)\mathrm{g}_0\,(\eta) - \mathrm{g}^2{}_{0\eta}$$
$$\mathrm{g}_{1\eta}(0) = 0, \quad \mathrm{g}_{1\eta}(\eta_\infty) = 0. \tag{25}$$

Here, we have considered only zeroth and first order equations. The zeroth order equations are in the form of confluent hypergeometric function defined by Abramowitz and Stegun (1965)

$$M(a,\,b\,;z) = 1 + \sum_{n=1}^{\infty} \frac{(a)_n z^n}{(b)_n n!},$$

where M is the Kummer's function and

$$(a)_n = a(a+1)(a+2)\,.........\,(a+n-1)$$
$$(b)_n = b(b+1)(b+2)\,.........\,(b+n-1).$$

Hence,

$$\theta_0(\eta) = C_1\, e^{-\alpha B_{21}\eta}\;M\left[B_{21}-2,\; 1+B_{21};\; -\frac{P_r}{\alpha^2}e^{-\alpha\eta}\right]$$
$$g_0(\eta) = C_2\, e^{\alpha B_{21}\eta}\;M\left[B_{21}-2,\; 1+B_{21};\; -\frac{P_r}{\alpha^2}\,e^{-\alpha\eta}\right]$$

where

$$C_1 = \frac{1}{M\left[B_{21}-2,\quad 1+B_{21};\quad \frac{-P_r}{\alpha^2}\right]}\quad,\; B_{21} \;=\; \frac{P_r}{\alpha^2} - \frac{P_r}{\sqrt{b\nu}}\frac{v_w}{\alpha}$$

and

$$C_2 = \frac{1}{M\left[B_{21}-2,\quad 1+B_{21};\quad \frac{-P_r}{\alpha^2}\right] + \left(\frac{B_{21}-2}{1+B_{21}}\right) M\left[B_{21}-1,\quad 2+B_{21};\quad \frac{-P_r}{\alpha^2}\right]}$$

Numerical solution

Since equations (23) and (25) are non-homogeneous ordinary differential equations with variable coefficients and we follow most efficient numerical shooting technique with fourth order Runge-kutta algorithm to solve them. In this method it is most important to choose the appropriate finite value of η_∞. To select η_∞ we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $\theta_{1\eta}(0)$. The solution process is repeated with another larger value of η_∞ until two successive values of $\theta_{1\eta}(0)$ differ only after desired significant digit. The last value of η_∞ is chosen as appropriate value for a particular set of parameters. For different set of parameters the appropriate values of η_∞ are different and they are obtained by the above procedure only. It is noticed that small values of P_r makes η_∞ larger. Blowing also gives large value for η_∞. Similarly we obtain the appropriate value of η_∞ for equation (25).

In the shooting technique we apply initial value method for which two more initial conditions are necessary. Since $\theta_{1\eta}(0)$ and g_1 (0) are not prescribed, we start with an initial approximation to the two unknowns $\theta_{1\eta}(0) = \alpha$ and g_1 (0) = β. For approximate values of η_∞ we consider two sets of initial approximations by choosing $\alpha = \alpha_0, \alpha_1$ and $\beta = \beta_0, \beta_1$, as $\alpha_0 = 1.0$,$\alpha_1 = -1.0$, $\beta_0 = -1.0$ $\beta_1 = 1.0$ (Chaim, 1998; Conte and de Boor, 1986). Then, we solve the problem using fourth order Runge-Kutta method for a general second order equation

$$\theta_{1\eta\eta} = f_1\ (\eta,\ \theta_1,\ \theta_{1\eta}) \qquad \text{with}\ \ \theta_1\,(0) = 0 \qquad \theta_{1\eta}(0) = \alpha_k$$
$$g_{1\eta\eta} = f_2\ (\eta,\ g_1,\ g_{1\eta}) \qquad \text{with}\ \ g_1\,(0) = \beta_k \qquad g_{1\eta}(0) = 0,$$

from $\eta = 0$ to $\eta = \eta_\infty$ and we call these solutions z $(\alpha_k,\ \eta_\infty)$ at $\eta = \eta_\infty$. Next approximation $\alpha_{k+1},\ \beta_{k+1}$ are obtained from the linear interpolations

$$\delta_{k+1} = \delta_{k-1} - (\delta_k - \delta_{k-1})\ \frac{z\ \ (\delta_{k-1}\ \ ,\eta_\infty)}{z\ (\delta_k,\ \ \eta_\infty)\ \ - z\ (\delta_{k-1},\ \eta_\infty)},$$

where $\delta_k = \alpha_k,\ \beta_k$

$$z = \theta_1, g_1 \qquad k = 1, 2, 3\ldots$$

We repeat the process of solution and linear interpolation incorporating the latest approximation until we get $|z\,(\delta_k,\ \eta_\infty) - 0\,| \ < \varepsilon$ for a prescribed ε.

Results and discussion

Numerical computation of the result has been carried out for various values of visco-elastic parameter (k_1), porosity parameter (k_2), in case of suction, blowing and impermeability of the boundary wall. Values of perturbation parameter ε is chosen very small. Results are computed for small values of Prandtl number, which is applicable to the case of polymer solution. Kinematic viscosity v is taken as 0.04 and stretching rate b=2.0. Results are depicted in Figures 2-8. Non-dimensional horizontal velocity profiles are shown in Figures 2(a) and (b) for different values of porosity parameter. Effect of suction, blowing and impermeability of the wall on shear stress has been shown graphically in

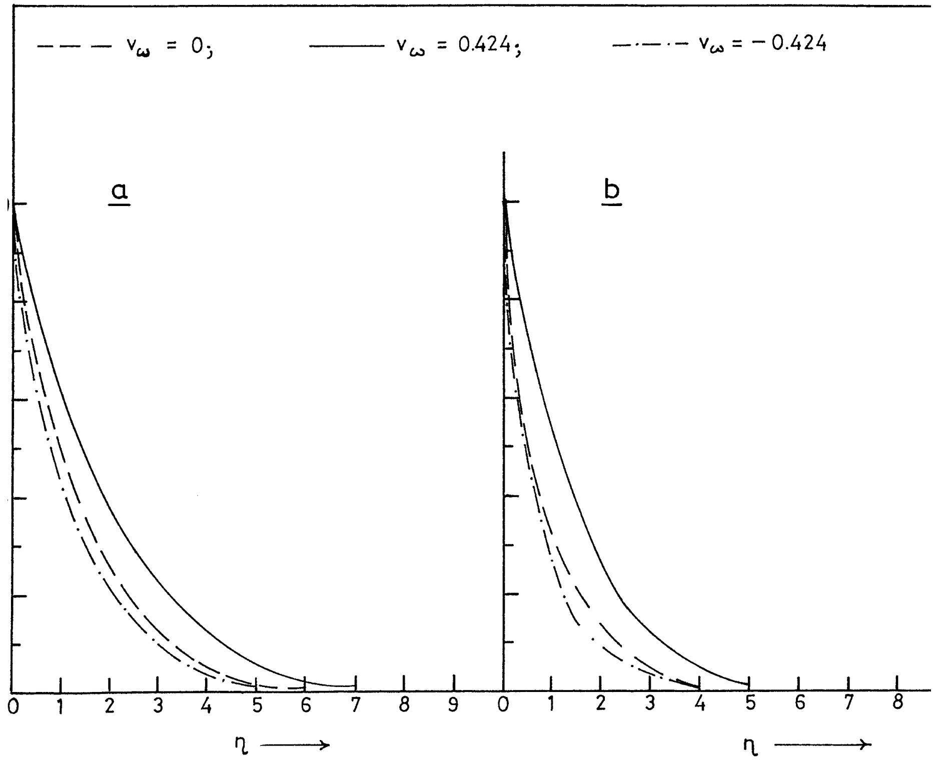

Figure 2.
Velocity profile $f'(\eta)$ in absence of porous medium when (a) k_1 - 0.04, $k_2 = 0$ and (b) $k_1 = 0.04$, $k_2 = 1.0$

Figures 3(a) and (b) for various values of porosity parameter. Results for prescribed surface temperature (PST) are drawn in Figure 4-6 and results for prescribed heat flux (PHF) are presented in Figures 7 and 8.

The effects of impermeability of the boundary wall, suction and blowing on the horizontal velocity profiles in the boundary layer in absence of porous medium are shown in Figure 2(a). It is observed that velocity decreases in the boundary layer with the increase of distance from the boundary. The effect of suction is to decrease the velocity and that of blowing is to increase the velocity. These results are consistent with the physical situation. Figure 2(b) is plotted for same set of parameters except for non-zero values of porosity parameter ($k_2 = 1.0$). Comparison of these two graphs reveals that the effect of porosity is to decrease the velocity for all cases of suction, blowing and impermeability of the wall. This is because of porous medium's obstruction to the flow over the sheet. Figure 3(a) and (b) display shear stress contour on the boundary against the visco-elastic parameter k_1 for different values of porosity parameter k_2. It is noticed that the magnitude of shear stress is small on the boundary for blowing in comparison to the case of suction. This is due to the fact that injection of the fluid into the boundary layer amounts to the increase of fluid velocity, resulting in reduction of frictional force. Comparison study of

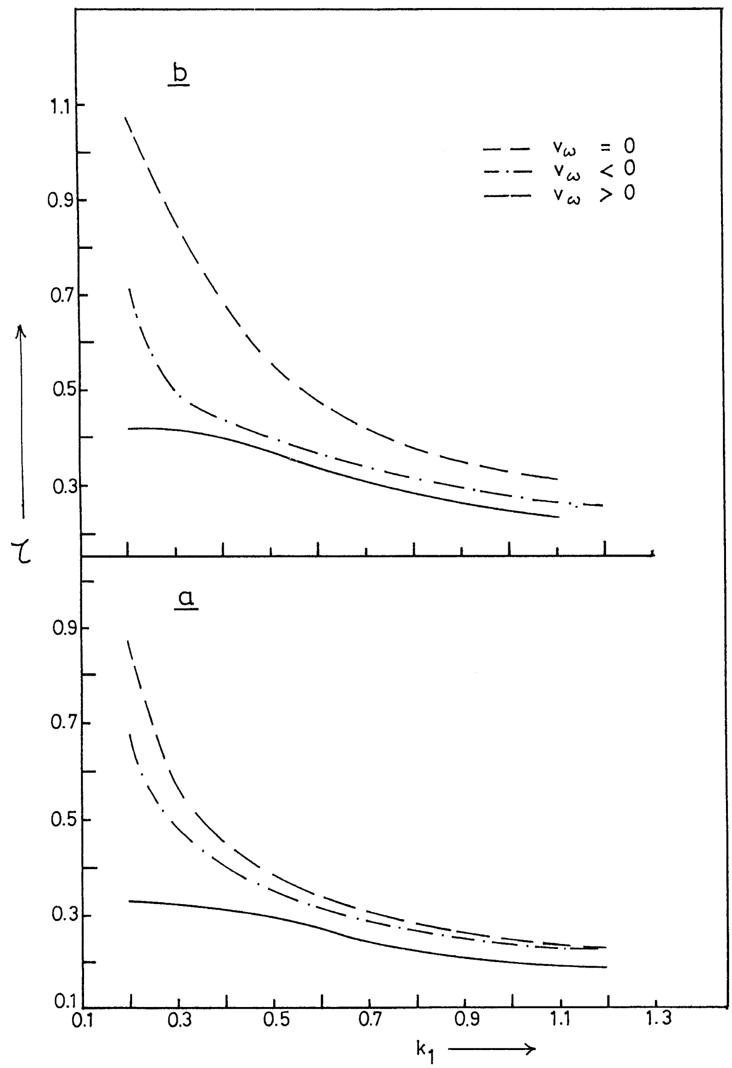

Figure 3.
Graph of shear stress vs visco-elastic parameter k_1 for various values of suction, blowing and impermeability of the wall when (a) $k_2 = 0$ and (b) $k_2 = 1.0$

Figures 3(a) and (b) shows that porosity introduces additional shear stress on the boundary. Limiting cases of our result when the wall sheet is impermeable leads to the result of Abel and Veena (1998).

A graph is plotted in Figure 4 for wall temperature gradient $-\theta_\eta(0)$ versus ε for all cases of suction ($v_w = -0.235$), blowing ($v_w = 0.424$) and impermeability ($v_w = 0.0$) of the wall when Prandtl number is $P_r = 1.0$. It is interesting to notice that the effect of visco-elastic parameter is not significant in case of impermeability of the wall, whereas the effect of visco-elasticity in the case of suction is significant and it increases the wall temperature gradient $-\theta_\eta(0)$ in case of suction. However, the porosity has significant impact on the wall

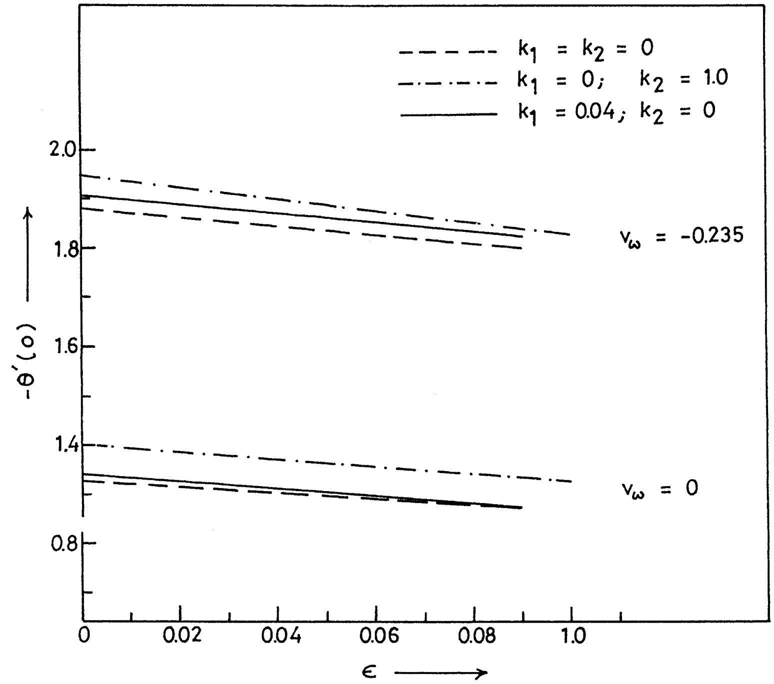

Figure 4.
Dimensionless wall temperature gradient, $-\theta'(0)$, for different values of k_1 and k_2 in PST case when Pr = 1.0

temperature gradient $-\theta_\eta(0)$ in all cases. The effect of porosity is to increase the wall temperature gradient $-\theta_\eta(0)$ in the case of suction and also in the case when the stretching sheet is impermeable.

Figure 5 is the graph of wall temperature gradient $-\theta_\eta(0)$ versus ε for various values of Prandtl number for visco-elastic parameter $k_1 = 0.04$ and porosity parameter $k_2 = 1.0$. From the graphical representation we analyse that the effect of increasing Prandtl number is to increase the wall temperature gradient $-\theta_\eta(0)$ in case of suction and also in the case when the stretching sheet is impermeable. This is due to the fact that thermal boundary layer thickness decreases with the increase of Prandtl number. For a fixed value of P_r the value of wall temperature gradient $-\theta_\eta(0)$ in case of suction is larger than that in the case of blowing. This is due to the fact that the thermal boundary layer thickness, in case of suction, is thinner than that in the case of blowing. Also, this graph shows the combined effect of visco-elasticity and porosity on the wall temperature gradient. Comparison of Figure 4 and Figure 5, reveals that the combined effect of visco-elasticity and porosity is to decrease the wall temperature gradient $-\theta_\eta(0)$ in case of blowing and to increase in the case of stretching of the wall subjected to suction.

Figure 6 shows temperature profiles $\theta(\eta)$ versus space variable η for various values of visco-elastic parameter k_1. It is of some interest to note that the increase of visco-elastic parameter k_1 leads to the decrease of temperature profile in all cases of suction, blowing and impermeability of the wall.

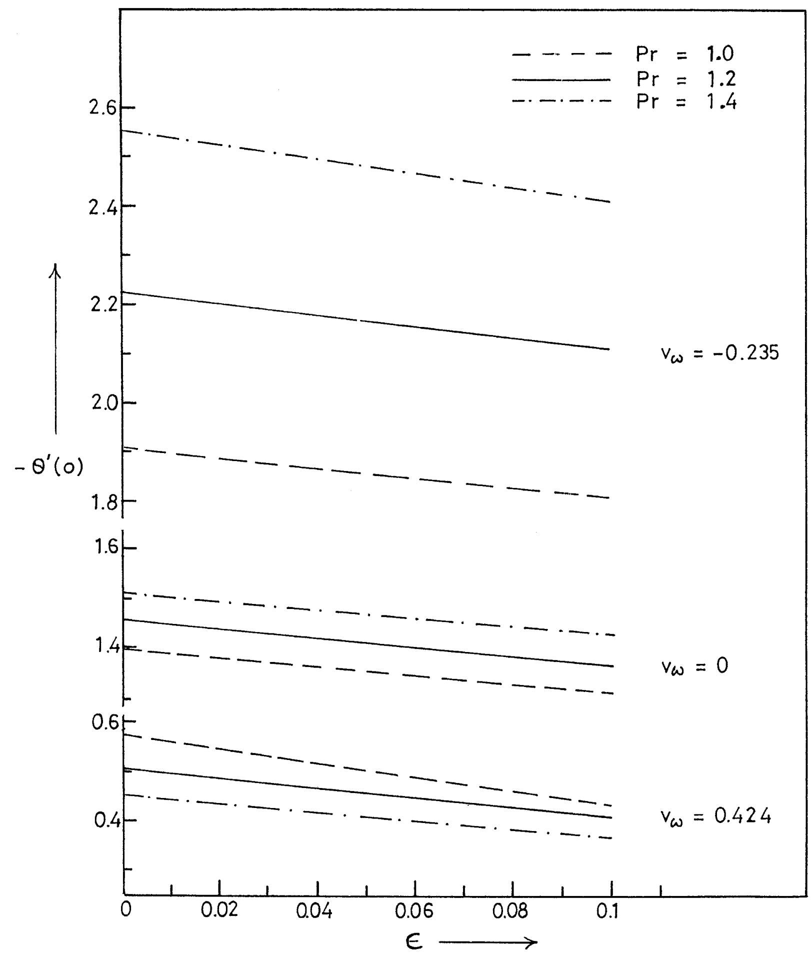

Figure 5.
Dimensionless wall temperature gradient $-\theta'(0)$ for different values of Pr in PST case (k_2 = 1.0; Pr = 1.0)

Figures 7(a) and (b) describe relationship of the non-dimensional temperature g(0) with ε in all cases of suction, blowing and impermeability of the wall when the Prandtl number P_r = 1.0. Figure 7(a) demonstrates that the effect of increasing porosity is to decrease the wall temperature in the case of suction and impermeability of the wall, but it is to increase wall temperature in the case of blowing. However, the effect of porosity has significant impact on the wall temperature gradient g(0) in both the cases of blowing and impermeability of the wall. Figure 7(b) shows that visco-elastic parameter k_1 has similar effect as that of porosity on temperature profile. It is also seen that the combined effect of porosity (k_2) and visco-elasticity (k_1) is to magnify the impact on wall temperature in PHF case.

Figure 8 gives the effect of Prandtl number on the dimensionless wall temperature for all cases of suction, blowing and impermeability of the wall.

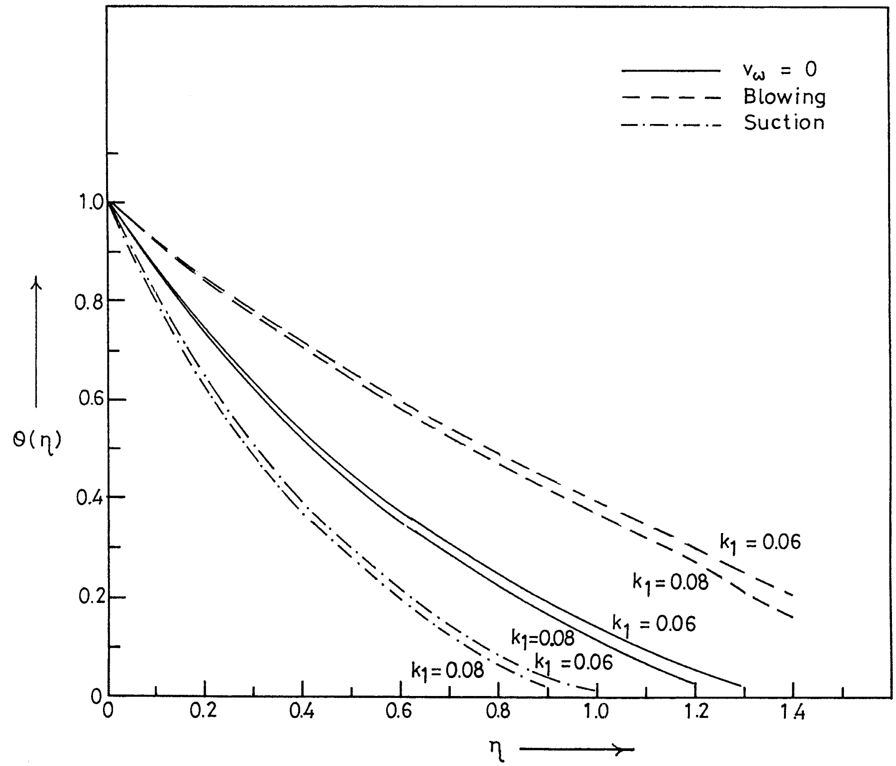

Figure 6.
Dimensionless temperature profile $\theta\eta$ vs η for different values of k_1 in PST case (k_2 = 1.0)

We notice that increase of Prandtl number reduces the wall temperature in all the cases of suction, blowing and impermeability of the wall.

Conclusions

The important findings of our study are as follows:

(1) The effect of suction is to decrease the velocity and that of blowing is to increase the velocity in the flow field. The effect of porosity is to decrease the velocity in the boundary layer in both the cases of blowing and suction. In order to validate our numerical values, we have compared horizontal velocity profiles in our case for an impermeable stretching sheet (v_w = 0) with those of Abel and Veena (1998). The results are found to be in very good agreement.

(2) The effect of porosity is to increase the wall temperature gradient $-\theta_\eta(0)$ when there is suction and impermeability of the wall.

(3) The effect of increasing visco-elastic parameter is to decrease the temperature profile.

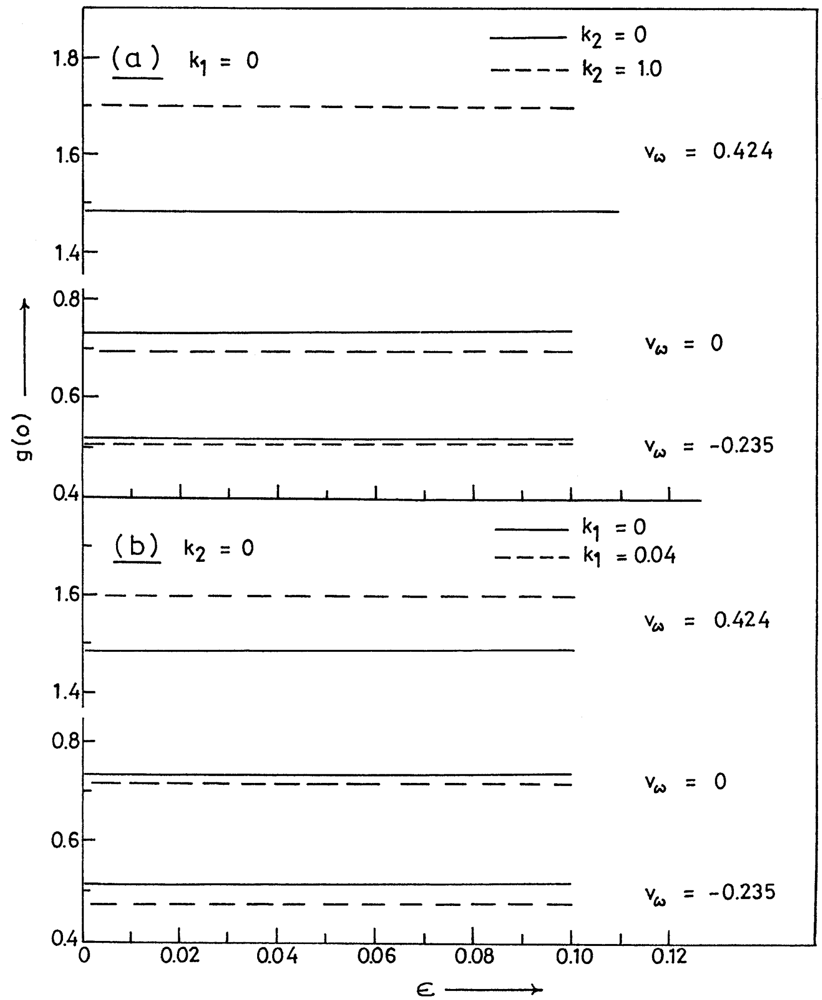

Figure 7.
Dimensionless wall temperature, g (0), vs ε for different values of (a) k_2 and (b) k_1 in PHF case when Pr = 1.0

(4) Increase of Prandtl number results in decrease of wall temperature in the case of suction, blowing and also in the case when the stretching sheet is impermeable.

(5) The combined effect of visco-elasticity and porosity is to increase the wall temperature in case of blowing and to decrease in case of suction and when the stretching sheet is impermeable.

It should noted that some of the published results (Chen and Char, 1988; Chaim, 1996; Gupta and Sridhar, 1985; Abel and Veena, 1998) may be obtained as special cases of the present work.

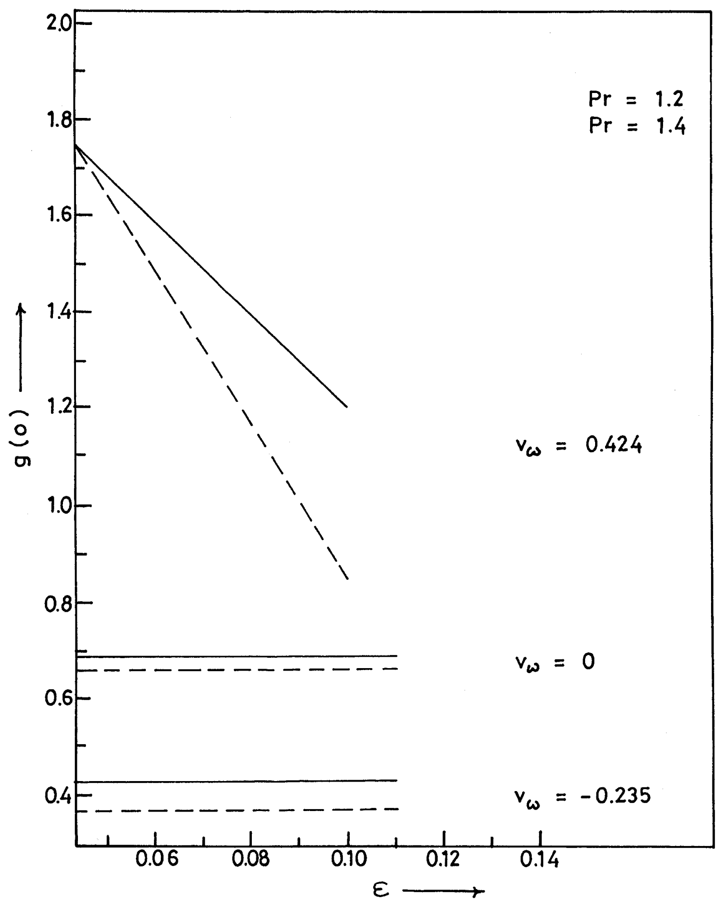

Figure 8.
Dimensionless wall temperature, g(0), vs ε for different values of Pr in PHF case ($k_1 = 0.04$, $k_2 = 1.0$)

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